Notes for Lab # 3

Review of RC Circuits:

Energy storage devices such as capacitors and inductors can be used to filter out undesired frequencies due to their frequency dependent impedance. This is the basic principle that allows us to tune into different radio stations, increase the bass or treble on your stereo, and transmit vast amounts of information over a single wire!

Figure 3-1 shows a basic RC circuit. The easiest way to solve for these circuits is to put the impedances into the Laplace domain and solve for the ratio of the output voltage divided by the input voltage. This is also called the gain. We are usually also concerned with the phase but we will not discuss that in this lab.

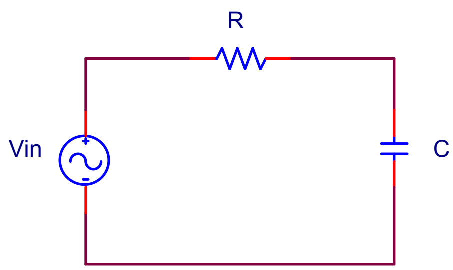

Figure 3-1: A simple RC circuit. The output voltage has not yet been defined and can be taken over either the capacitor or the resistor.

The output voltage can be defined as either the voltage across the resistor (V_R) or the voltage across the capacitor (V_C), each output will yield different results. We will take a look at the voltage coming off the resistor.

Step1: Represent the impedances in the Laplace domain.

$$R \rightarrow R$$
$$C \rightarrow 1/j\omega C$$

Step2: The output voltage is simply a voltage division of the input voltage

$$V_R = V_{in} \cdot \frac{R}{R + \frac{1}{j\omega C}}$$

Step3: If we pretty up the equation we can have a better idea of what type of filter we have.

$$\frac{V_R}{V_{in}} = \frac{R \cdot j\omega C}{R \cdot j\omega C + 1}$$

In order to figure out what type of filter we have we can look at what happens to the output at low frequencies and then at high frequencies. When ω approaches zero, we have the output approach zero. As ω approaches infinity, the output approaches one. Therefore, we have a high pass filter. Low frequencies are suppressed while higher frequencies are allowed to pass.

Review of the -3dB Point:

As discussed above, different frequencies are filtered since the capacitor applies a different impedance based on a given frequency. That means that there is no clear cut off where one frequency is rejected while another accepted. We therefore must define a standard to define a filters cut off point. We generally use what's called the 3dB point, or -3dB point. You have probably herd of it before. The 3dB point is the point in which your output has half the power as the maximum output. The term -3dB is just the log representation of the number ½ , $10\log_{10}(1/2)$. This standard is used in other applications as well, such as televisions. When a television says it has a 5 year life, this does not mean that in 5 years it will break, it means that in 5 years the television will be ½ as bright as it was the day it was first used.

Now let's look at our previous example and find the -3dB point. The maximum output of our above example was a gain of 1. In the log domain, a gain of 1 is 0dB. Hence, the -3dB point will be 0dB – 3dB = -3dB. One important thing to note is that the dB scale is a representation of power while our output is in terms of voltage. This can be fixed by recognizing that voltage squared is proportionate to power. Let's now go ahead and solve for out -3dB point.

First, we need to change our output equation into the standard form. This form is standard because it helps us see a simple solution as we will see later.

$$\frac{V_R}{V_{in}} = \frac{R \cdot j\omega C}{R \cdot j\omega C + 1} = \frac{1}{1 + \frac{1}{R \cdot j\omega C}} = \frac{1}{1 + \frac{1/RC}{j\omega}}$$

We are concerned with the gain, which is the magnitude of $\frac{V_{out}}{V_{in}}$. So we want

$10\log_{10}(\left|\frac{V_R}{V_{in}}\right|^2) = -3dB$, which, as shown before means that $\left|\frac{V_R}{V_{in}}\right|^2 = \frac{1}{2}$.

$$\left|\frac{1}{1 + \frac{1/RC}{j\omega}}\right|^2 = \frac{1}{\left|1 + \frac{1/RC}{j\omega}\right|^2} = \frac{1}{\left|1 - j \cdot \frac{1/RC}{\omega}\right|^2} = \frac{1}{2}$$

To solve this, first note that $|1 \pm 1j| = \sqrt{2}$, therefore, solution occurs when $\omega = {}^{1}\!/_{RC}$

Therefore, the -3dB point for an RC filter is found at $f = \frac{1}{2\pi RC}$.

Review of Op-Amp Circuits:

There are many different ways to utilize an Operational Amplifier (Op-Amp), today we will be looking at a simple inverting amplifier. Figure 3-2 shows a basic example of an inverting op-amp. We will be solving for the gain of the op-amp using nodal analysis. We know that from the properties of an op-amp, the voltage going into the positive and negative inputs must be the same. Hence, V_A must be equal to zero volts. Below we use nodal analysis at the point V_A to solve for the gain.

$$\frac{0 - V_{in}}{R_1} + \frac{0 - V_{out}}{R_2} = 0$$

$$\frac{-V_{in}}{R_1} = \frac{V_{out}}{R_2}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

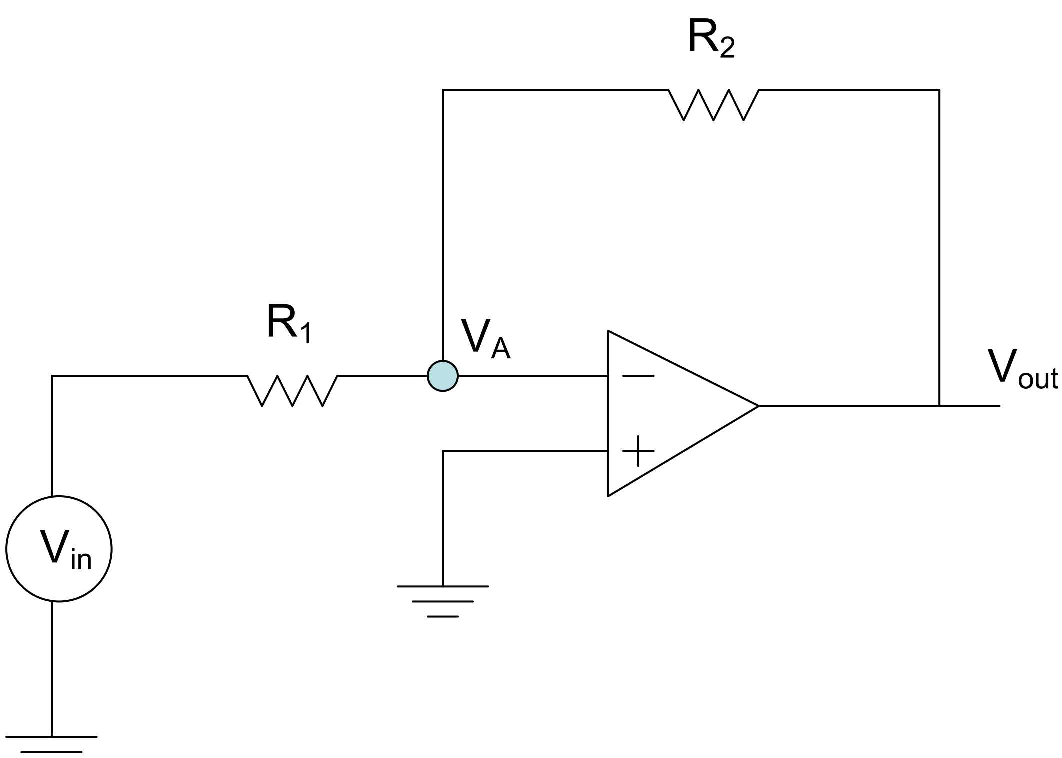

Figure 3-2: A simple inverting op-amp circuit