Chapter II-Lecture 9 of 10

Transmission Lines - Wave Equations

o ***The Admittance Smith Chart***
o ***Smith Chart Features and Short Cuts***

The Admittance Smith chart:

At high frequencies, it is often preferred to work with admittances instead of impedances. One reason is that most circuit components are mounted in parallel with one terminal grounded. Series components are floating devices and would lead to the circuit suffering from additional undesired parasitic. Working with shunt devices makes admittance expressions more attractive since combining admittances in parallel is achieved through simple addition of their values.

Examining expression (2.101), we can rewrite it in terms of admittance components as follows:

$$Z_n(z\ or\ d) = \frac{Z(z\ or\ d)}{Z_o} = \frac{[1+\Gamma(z\ or\ d)]}{[1-\Gamma(z\ or\ d)]}\ \ and\ \ \Gamma(z\ or\ d) = \frac{[Z_n(z\ or\ d)-1]}{[Z_n(z\ or\ d)+1]} \qquad (2.101)$$

$$Y_n(z\ or\ d) = \frac{Y(z\ or\ d)}{Y_o} = \frac{1}{Z_n(z\ or\ d)} = \frac{[1-\Gamma(z\ or\ d)]}{[1+\Gamma(z\ or\ d)]}\ \ and\ -\Gamma(z\ or\ d) = \frac{[Y_n(z\ or\ d)-1]}{[Y_n(z\ or\ d)+1]} \qquad (2.111)$$

Comparing Equations (2.101) and (2.111), we notice the similarity except for replacing the Z_n by Y_n and at the same time replacing Γ by $-\Gamma$. Likewise, if we express Y_n in terms of its real and imaginary conductance g_n and susceptance b_n, $Y_n = g_n + jb_n$, we can show that the constant g and b circles are of the form:

$$\left[-\Gamma_{re} - \frac{g}{1+g}\right]^2 + [-\Gamma_{im}]^2 = \left[\frac{1}{1+g}\right]^2 \qquad (2.112)$$

$$[-\Gamma_{re} - 1]^2 + \left[-\Gamma_{im} - \frac{1}{b}\right]^2 = \left[\frac{1}{b}\right]^2 \qquad (2.113)$$

which are, again, similar to (2.102) and (2.103) by replacing r and x by g and b and at the same time replacing Γ_{re} and Γ_{im} by $-\Gamma_{re}$ and $-\Gamma_{im}$, respectively.

The conclusion is that for a given Γ phasor on the chart (corresponding to a given value of Z_n=r+jx), the $-\Gamma$ phasor on the same chart corresponds to the Y_n=g+jb value at that location on the line.

This concept is demonstrated in the chart of Figure 2.47. The Γ phasor is shown for the normalized impedance Z_n=01.5+j2.0. Thus, the corresponding $-\Gamma$ phasor reveals to us the corresponding Y_n which is Y_n=0.24-j0.32. (It is interesting to observe that the chart can be used as a complex number inverter since $Y_n = 1/Z_n$. Hence, if you place a complex number as the Z_n, the corresponding Y_n point is the inverse of that complex number.)

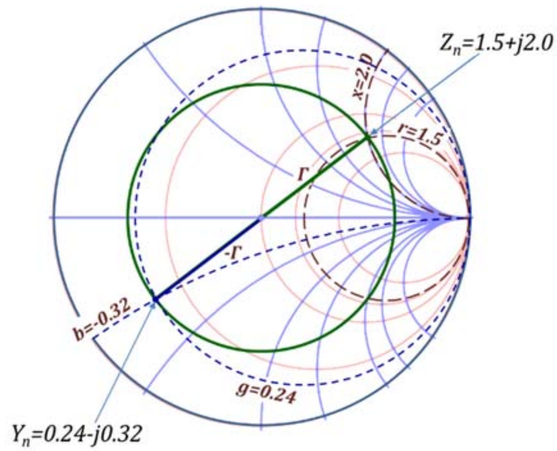

Figure 2.47

Now that we identified the placement of admittances on the chart, we can actually work out series and parallel combinations in conjunction with segments of transmission lines in a relatively convenient way. To demonstrate, let us consider the example of Figure 2.48.

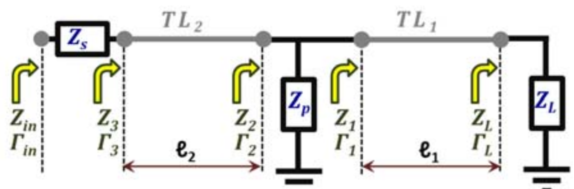

Figure 2.48

The figure shows a two-transmission line segments TL_1 and TL_2, both having the same Z_o and γ but different lengths. The two lines are connected in cascade with a shunt impedance Z_p in between. TL_1 is terminated in Z_L while a series impedance Z_s is connected at the input side of TL_2. It is desired to find the combined circuit input impedance Z_{in} and reflection coefficient phasor Γ_{in}.

To solve this problem analytically, we would start with TL_1 terminated in Z_L. Using Equation (2.71), we obtain the input impedance to that segment, Z_1. Next, we add Z_p in parallel to Z_1 to get Z_2. Again, we use Equation (2.71) for TL_2 terminated in Z_2 as a load and we get its input impedance Z_3, and finally, add Z_s in series to get Z_{in}. To get Γ_{in}, we use Equation (2.46).

To do this problem using the Smith Chart, we follow the procedure outlined below and demonstrated in Figure 2.49.

1. Choose the normalization impedance to be Z_o of the TLs.
2. Compute the normalized $Z_{Ln}= Z_L/Z_o$
3. Locate Z_{Ln} on the chart and identify the Γ_L phasor point [1].
4. Draw a circle centered at the origin passing by Γ_L
5. Move on the circle of step 4 above "towards generator" a normalized distance of ℓ_1/λ. The new point represents Γ_1 [2]. You may read Z_{1n} and Γ_1 from this point.
6. Extend the Γ_1 phasor through the origin to reach the $-\Gamma_1$ point [3]. This is the Y_{1n} point. You may read Y_{1n} off this point.
7. Compute Y_{pn} as $1/Z_{pn}$ (or Z_o/Z_p).

8. Add Y_{pn} to Y_{1n} to obtain Y_{2n}. Locate Y_{2n} on the chart [5]. You may read $-\Gamma_2$ off that point.
9. Draw a circle centered at the origin passing by $-\Gamma_2$.
10. Extend the $-\Gamma_2$ phasor through the origin to reach the $+\Gamma_2$ point [6]. This is the Z_{2n} point. You may read Z_{2n} and Γ_2 off this point.
11. Move on the circle of step 9 above "towards generator" a normalized distance of ℓ_2/λ. The new point represents Γ_3 [7]. You may read Z_{3n} and Γ_3 from this point.
12. Compute Z_{sn} as Z_s/Z_o.
13. Add Z_{sn} to Z_{3n} to get Z_{in}. Locate Z_{in} on the chart and identify the Γ_{in} phasor point [9]. You may read Z_{inn} and Γ_{in} from this point.
14. De-normalize Z_{inn} to obtain $Z_{in}=Z_o*Z_{inn}$.

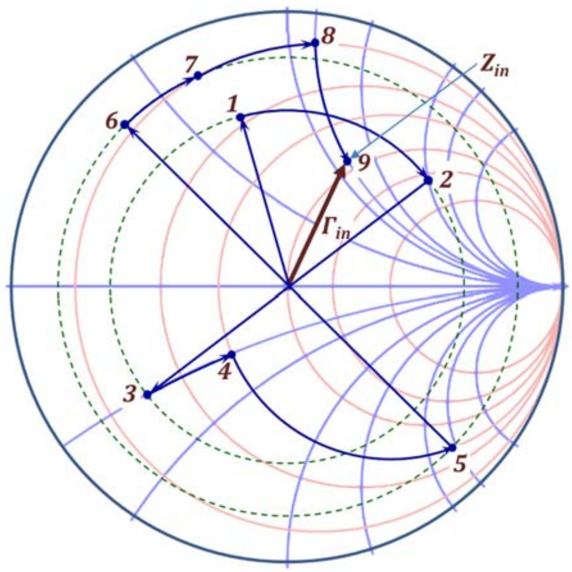

Figure 2.49

It is worth noting that the impedance and admittance additions in steps 8 and 13 above can be performed on the chart. This is simply done by the imaginary part first then the real part (or vice-versa). To add the imaginary part, you move on a constant real part circle the proper amount that corresponds to the added imaginary part. Likewise, to add the real part, you move on a constant imaginary part circle the proper amount that corresponds to the added real part. This is demonstrated in Figure 2.49 in the steps 3-4, 4-5, 7-8, and 8-9.